Math 115

Spring 2017

Lecture 12

we are given two Points $A(-4,2)$, $B(0,5)$

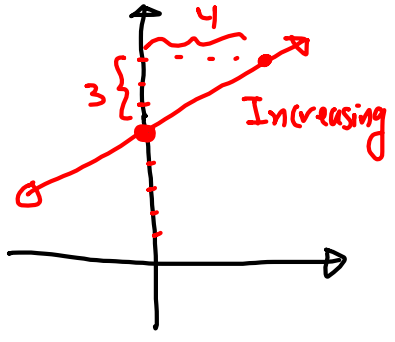
① find the slope of line $\overleftrightarrow{AB}$

$$m = \frac{2-5}{-4-0} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$$

② find the equation of the line $\overleftrightarrow{AB}$

$y - 5 = \frac{3}{4}(x-0)$ $\quad y - 5 = \frac{3}{4}x$ $\quad \boxed{y = \frac{3}{4}x + 5}$

③ Draw the line $\overleftrightarrow{AB}$.



$A(-5,-3)$, $B(3,3)$

① find the distance from A to B.

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(-5-3)^2+(-3-3)^2}=\sqrt{(-8)^2+(-6)^2}$$
$$=\sqrt{64+36}=\sqrt{100}=\boxed{10}$$

② find the midpoint of $\overline{AB}$.

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{-5+3}{2},\frac{-3+3}{2}\right)=M\left(\frac{-2}{2},\frac{0}{2}\right)=M(-1,0)$$

③ find the slope of line $\overleftrightarrow{AB}$

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{-3-3}{-5-3}=\frac{-6}{-8}=\boxed{\frac{3}{4}}$$

④ find eqn of the line $\overleftrightarrow{AB}$

$$y-y_1=m(x-x_1)$$
$$y-3=\frac{3}{4}(x-3)$$
$$y-3=\frac{3}{4}x-\frac{9}{4}$$
$$y=\frac{3}{4}x-\frac{9}{4}+3$$
$$y=\frac{3}{4}x-\frac{9}{4}+\frac{12}{4}$$
$$\boxed{y=\frac{3}{4}x+\frac{3}{4}}$$

find eqn of a line that contains $(5,-3)$ with

a) Zero slope
Horizontal line
$y=b$ $\boxed{y=-3}$

b) No slope
Vertical line
$x=a$ $\boxed{x=5}$

c) undefined slope
Vertical line
$x=a$ $\boxed{x=5}$

d) Slope $\frac{2}{5}$

$$y-y_1=m(x-x_1)$$
$$y-(-3)=\frac{2}{5}(x-5)$$
$$y+3=\frac{2}{5}x-\frac{2}{5}\cdot 5$$
$$y+3=\frac{2}{5}x-2$$
$$\boxed{y=\frac{2}{5}x-5}$$

Find eqn of line that contains $(3, -4)$ and is **parallel** to the line $2x - 3y = 9$.

$y - y_1 = m(x - x_1)$

$y - (-4) = \frac{2}{3}(x - 3)$

$y + 4 = \frac{2}{3}x - \frac{2}{3} \cdot 3$

$y = \frac{2}{3}x - 6$

Same slope

$-3y = -2x + 9$

$y = \frac{-2}{-3}x + \frac{9}{-3}$

$y = \frac{2}{3}x - 3$

are parallel lines

Find eqn of aline that contains $(-2, 5)$ and is **perpendicular** to the line $y = -2x - 3$.

Graph both lines.

$m = -2$

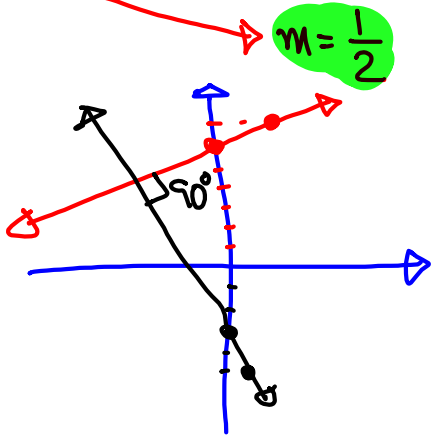
$m = \frac{-2}{1}$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{2}(x - (-2))$

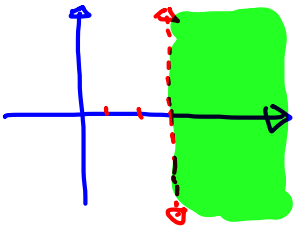
$y - 5 = \frac{1}{2}x + \frac{1}{2} \cdot 2$

$y = \frac{1}{2}x + 6$

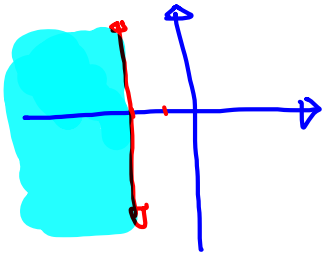


Graph & shade:

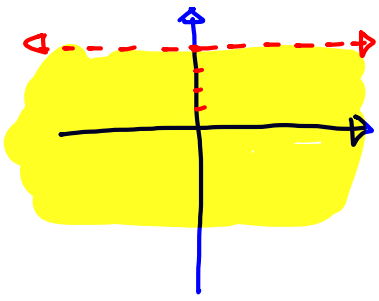
① $x > 3$



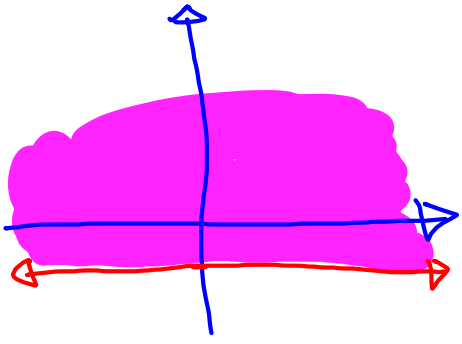
② $x \leq -2$



③ $y < 4$

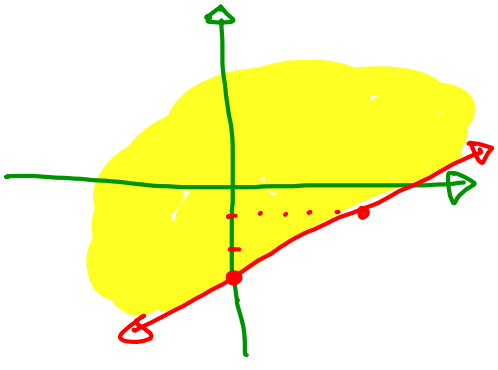


④ $y \geq -1$

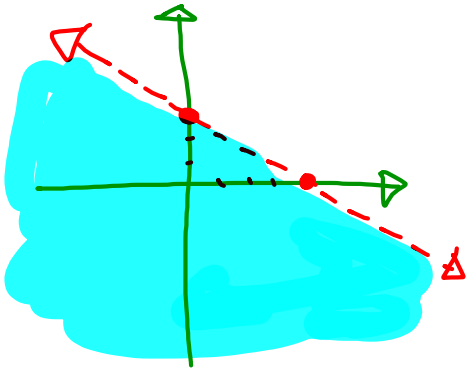


Graph & shade

① $y \geq \frac{2}{5}x - 3$



② $y < \frac{-3}{4}x + 3$



Graph & Shade:

$2x - 3y \leq 9$

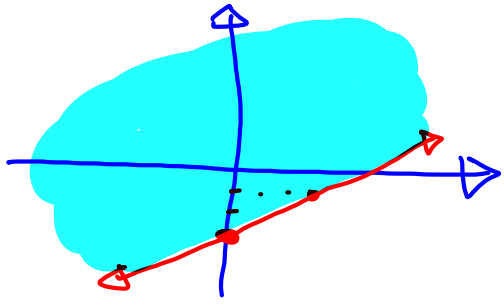
Hint: write in Slope-Int. form "Isolate y".

$-3y \leq -2x + 9$

Divide by -3

$\frac{-3}{-3}y \geq \frac{-2}{-3}x + \frac{9}{-3}$

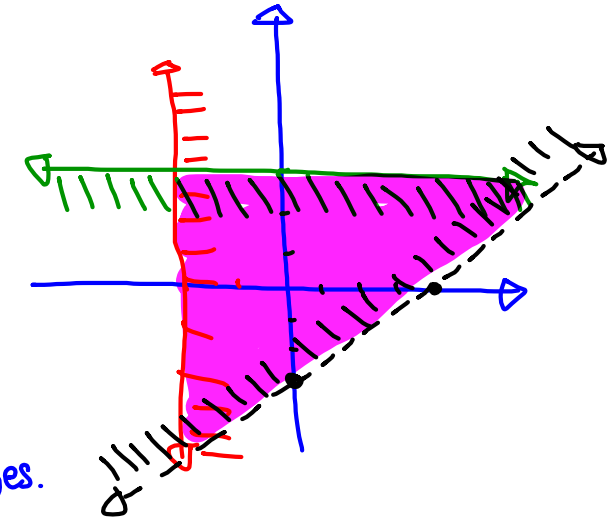
$y \geq \frac{2}{3}x - 3$



Graph & Shade

$$\begin{cases} x \geq -2 \\ y \leq 3 \\ y > \frac{3}{4}x - 3 \end{cases}$$

Only shade the region that is shaded for all 3 edges.



Ch. 8 : System of linear equations in Two variables.

$\begin{cases} x+y=4 \\ x-y=2 \end{cases}$, $\begin{cases} 3x-2y=5 \\ y=x+3 \end{cases}$, $\begin{cases} 4x+3y=8 \\ 2x-4y=7 \end{cases}$

If we have a Soln, the solution is an ordered-pair (x, y) which satisfy both equations. for example:

Is $(-2,3)$ a Solution for $\begin{cases} 2x+3y=5 \\ x+y=-1 \end{cases}$?

$2x+3y=5$
$2(-2)+3(3)=5$
$-4+9=5$
$5=5$ ✓

$x+y=-1$
$-2+3=-1$
$1=-1$
false

$(-2,3)$ is not a Soln.

Is $(2,-5)$ a Solution for $\begin{cases} 3x+y=1 \\ 2x-3y=19 \end{cases}$?

Checking Eqn 1:
$3(2)+(-5)=1$
$6-5=1$
$1=1$ ✓

Checking Eqn 2:
$2(2)-3(-5)=19$
$4+15=19$
$19=19$ ✓

Yes $(2,-5)$ is a Soln.

Methods on Solving system of linear equations:

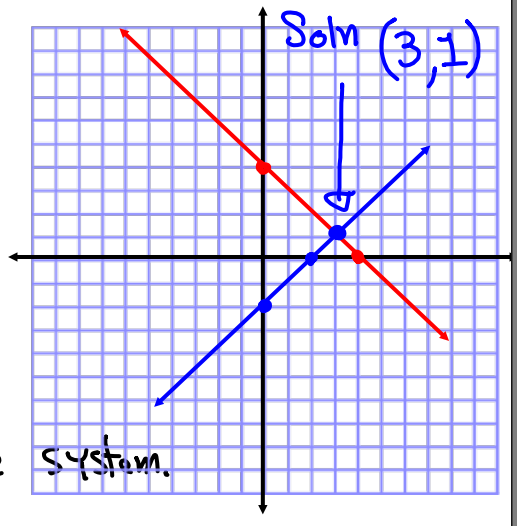
1) Graphing

2) Substitution

3) Addition / Elimination

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

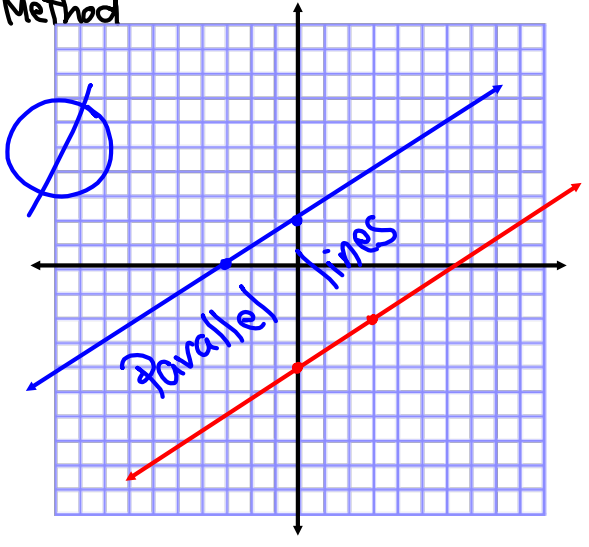
Graph both lines in the same coordinate system.



Solve by Graphing Method

$$\begin{cases} y = \frac{2}{3}x - 4 \\ 2x - 3y = -6 \end{cases}$$

x	y
0	2
-3	0



Solve by graphing:

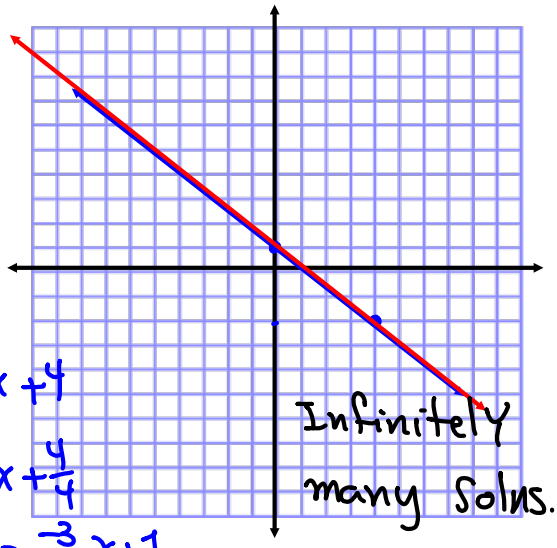
$$\begin{cases} y = \frac{-3}{4}x + 1 \\ 3x + 4y = 4 \end{cases}$$

x	y
0	1
4	-2

$4y = -3x + 4$

$y = \frac{-3}{4}x + \frac{4}{4}$

$y = \frac{-3}{4}x + 1$



Infinitely many Solns.

$$\begin{cases} 3x - 2y = -3 \\ y = x + 1 \end{cases}$$

Solve by Subs. method

$3x - 2(x+1) = -3$

$3x - 2x - 2 = -3$

$x = -1$

$y = -1 + 1$

$y = 0$

Final ans $(-1, 0)$

Solve by Subs. method.

$$\begin{cases} 4x - y = 5 \\ y = 3x - 2 \end{cases}$$

$4x - (3x - 2) = 5$

$4x - 3x + 2 = 5$

$\boxed{x = 3}$

$y = 3(3) - 2$

$\boxed{y = 7}$

Final Ans $(3, 7)$

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

Solve by addition/Elimination method.

$2x = 6 \Rightarrow \boxed{x = 3}$

$\Rightarrow (3, 1)$

Now $3 + y = 4 \Rightarrow \boxed{y = 1}$

$$\begin{cases} 3x + 2y = -10 \\ 2 \left(x - y = 0 \right) \end{cases}$$

$$\begin{cases} 3x + 2y = -10 \\ 2x - 2y = 0 \end{cases}$$

$$5x = -10$$

$$\boxed{x = -2}$$

$$-2 - y = 0$$

$$-2 = y$$

$$\boxed{y = -2}$$

Final Ans.

$(-2, -2)$.

$$\begin{cases} 2x + 3y = 6 \\ -2 \left(x + y = -4 \right) \end{cases} \Rightarrow \begin{cases} 2x + 3y = 6 \\ -2x - 2y = 8 \end{cases}$$

$$\boxed{y = 14}$$

$$x + 14 = -4$$

$$\boxed{x = -18}$$

Final Ans. $(-18, 14)$

Mark has $3.10 in nickels & Dimes only. The number of dimes is 1 more than twice the # of nickels. How many of each?

Nickels → x

Dimes → $2x+1$

$5x + 10(2x+1) = 310$

Value in Nickels | Value in Dimes | Total Value

$5x + 20x + 10 = 310$

$25x = 300$

$x = 12$

12 Nickels & 25 Dimes

N → Nickels

D → Dimes

$$\begin{cases} 5N + 10D = 310 \\ D = 2N + 1 \end{cases}$$

by Subs. method

$D = 2(12) + 1$

$D = 25$

$5N + 10(2N+1) = 310$

$5N + 20N + 10 = 310$

$25N = 300$

$N = 12$

12 Nickels & 25 Dimes

Maria has \$415 in \$5 bills and \$10 bills.

She has a total of 50 bills.

How many of each?

$x \rightarrow$ # \$5 bills,

$50-x \rightarrow$ # \$10 bills

$5x + 10(50-x) = 415$

$5x + 500 - 10x = 415$

$-5x = -85$

$\boxed{x = 17}$

17 \$5 bills
&
33 \$10 bills

Let $x \rightarrow$ # of \$5 bills

$y \rightarrow$ # of \$10 bills

$$\begin{cases} 5x + 10y = 415 \\ x + y = 50 \end{cases} \quad (-5)$$

$$\Rightarrow \begin{cases} 5x + 10y = 415 \\ -5x - 5y = -250 \end{cases}$$

$5y = 165$

$\boxed{y = 33}$

$x + 33 = 50$

$\boxed{x = 17}$

She has 17 \$5 bills
and 33 \$10 bills.

SG 9
Due tomorrow